$$= \begin{bmatrix} -0.06 \\ 0.33 \\ -0.18 \end{bmatrix} = 0.33 \begin{bmatrix} -0.18 \\ 1 \\ 0.54 \end{bmatrix}$$

$$\rightarrow A^{-1} X_2 = \begin{bmatrix} 0.37 & 0 & -0.12 \\ 0 & 0.33 & 0 \\ -0.12 & 0 & -0.37 \end{bmatrix} \begin{bmatrix} -0.18 \\ 1 \\ 0.54 \end{bmatrix}$$

$$= \begin{bmatrix} -0.12 \\ 0.33 \\ -0.17 \end{bmatrix} = 0.33 \begin{bmatrix} -0.36 \\ 1 \\ -0.51 \end{bmatrix}$$

$$\rightarrow A^{-1} X^3 = \begin{bmatrix} 0.37 & 0 & -0.12 \\ 0 & 0.33 & 0 \\ -0.12 & 0 & -0.37 \end{bmatrix} \begin{bmatrix} -0.36 \\ 1 \\ -0.51 \end{bmatrix}$$

$$= \begin{bmatrix} -0.07 \\ 0.33 \\ 0.22 \end{bmatrix} = 0.33 \begin{bmatrix} -0.21 \\ 1 \\ 0.66 \end{bmatrix}$$

$$\rightarrow A^{-1} X_4 = \begin{bmatrix} 0.37 & 0 & -0.12 \\ 0 & 0.33 & 0 \\ -0.12 & 0 & -0.37 \end{bmatrix} \begin{bmatrix} -0.21 \\ 1 \\ 0.66 \end{bmatrix}$$

$$= \begin{bmatrix} -0.14 \\ 0.33 \\ -0.22 \end{bmatrix} = 0.33 \begin{bmatrix} -0.42 \\ 1 \\ -0.66 \end{bmatrix}$$

Ques find the smallest eigen ~~vector~~ value in magnitude of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \text{ with } X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(Forward) **Newton's Interpolation formula**

The Newton's interpolation formula for the successive arrangement of a system can be written as

$$f(x) = f(x_0 + uh) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{\underline{|2}} \Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{\underline{|3}} \Delta^3 f(x_0) + \cdots + \frac{u(u-1)\ldots(u-n-1)}{\underline{|n}} \times \Delta^n f(x_0)$$

where $u = \frac{x - x_0}{h}$ (no of intervals)

Proof let us consider a func^n $f(x)$ such that it represent the continuous form of the func^n $f(x)$ w.r.to the coordinate 'h' for this the func^n $f(x)$ can be written as

$$f(x_0) = A_0 + A_1(x - x_0) + A_2(x - x_0)(x - x_0 - h) + A_3(x - x_0)(x - x_0 - 2h)(x - h) + \cdots + A_n(x - x_0)(x - x_0 - h) + [x - x_0 - (n-1)h]$$

where the value of x change as to the value of 'h' i.e

$x = x_0,\ x_0+h,\ x_0+2h, \ldots$

on replacing these values in eq^n (1) we get

At $x = x_0$

$$f(x_0) = A_0 \quad \text{---(2)}$$

At $x = x_0 + h$

$$f(x_0+h) = A_0 + A_1(x_0+h-x_0) + A_2 x_0$$

$$f(x_0+h) = A_0 + A_1 h$$

$$A_1 = \frac{f(x_0+h) - A_0}{h}$$

$$\boxed{A_1 = \frac{f(x_0+h) - f(x_0)}{h} = \frac{\Delta f(x_0)}{h}}$$

Similarly at $x = x_0 + 2h$

$$f(x_0+2h) = A_0 + A_1(\cancel{x_0}+2h-\cancel{x_0}) + A_2(x_0+2h-x)(\cancel{x_0}+2h-\cancel{x_0}-h) + A_3 x_0$$

$$= A_0 + A_1(2h) + A_2(2h)(\cdot h)$$

$$f(x_0+2h) = A_0 + 2hA_1 + 2h^2 A_2$$

$$A_2 = \frac{f(x_0+2h) - A_0 - 2hA_1}{2h^2}$$

$$A_2 = \frac{f(x_0+2h) - f(x_0) - \frac{2\cancel{h}\,\Delta f(x_0)}{\cancel{h}}}{2h^2}$$

$$\boxed{A_2 = \frac{\nabla^2 f(x_0)}{|\underline{2}\ h^2}}$$

|||ly,

$$\boxed{A_3 = \frac{\nabla^3 f(x_0)}{|\underline{3}\ h^3}}$$

$\vdots$

$$\boxed{A_n = \frac{\nabla^n f(x_0)}{|\underline{n}\ h^n}}$$

on replacing these values in eq^n no (1) we get

$$f(x_0) = x_0 + \frac{\Delta f(x_0)}{h}(x-x_0) + \frac{\nabla^2 f(x_0)}{|\underline{2}\ h^2}(x-x_0)(x-x_0-h)$$

$$+ \frac{\nabla^3 f(x_0)}{|\underline{3}\ h^3}(x-x_0)(x-x_0-2h) + \ldots$$

we know that

$$u = \frac{x-x_0}{h}$$

Then $x - x_0 = uh$

|||ly, $h(u-1) = (x-x_0-h)$

$\vdots$

on replacing these value in above eq^n we get -

$$f(x) = f(x_0) + \frac{\cancel{h}\,u\nabla f(x_0)}{\cancel{h}} + \frac{h^2 u(u-1)\nabla^2 f(x_0)}{|\underline{2}\ h^2}$$

$$+ \frac{\nabla^3 f(x_0)}{|\underline{3}\ h^3} u(u-1)(u-2) + \frac{\nabla^n f(x_0) h^n}{|\underline{n}\ h^n} u(u-1)(u-2) \ldots (u-(n-1))$$

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$$f(x) = f(x_0) + u\nabla f(x_0) + \frac{u(u-1)\nabla^2 f(x_0)}{\lfloor 2} + \frac{u(u-1)(u-2)\nabla^3 f(x_0)}{\lfloor 3}$$

$$+ \cdots + \frac{\nabla^n f(x_0)\, u(u-1) \cdots [u(n-1)h]}{\lfloor n}$$

This is required formula for the Newton interp. with equal interval.

Ques from the following table find the no. of students who obtained less than 45 marks?

Marks	Marks obtained
30-40	31
40-50	42
50-60	51
60-70	35
70-80	31

Sol^n following table represent the variation in marks obtained by the students.

Marks	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
less than 40	31				
		42			
			-9		
less than 50	73	51		-25	
			-16		37
less than 60	124	35		+12	
			-4		
less than 70	159				
		31			
less than 80	190				

Here $x = 45$

$x_0 = 40$

$h = 50 - 40 = 60 - 50 = 70 - 60$

(59)

$= 80 - 70 = 10$

Then $u = \frac{x - x_0}{h} = \frac{45 - 40}{10} = 0.5$

Then from Newton Interpolation formula we know that

$$f(x) = f(x_0 + uh) = f(x_0) + u\nabla f(x_0) + \frac{u(u-1)}{\lfloor 2}\nabla^2 f(x_0)$$

$$+ \frac{u(u-1)(u-2)}{\lfloor 3}\nabla^3 f(x_0) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\lfloor 2}\nabla^2 f(x_0)$$

$$+ \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{\lfloor 3}\nabla + \frac{u(u-1)(u-2)(u-3)}{\lfloor 3}\nabla^4 f(x_0)$$

$$= f(x_0) + \frac{1}{2}\nabla f(x_0) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\lfloor 2}\nabla^2 f(x_0)$$

$$+ \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{\lfloor 3}\nabla^3 f(x_0) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{\lfloor 4}\nabla^4 f(x_0)$$

$f(x_0) = 31 + \frac{1}{2} \times 42 - \frac{9}{8} - \frac{3}{48} \times 25 - \frac{15}{384} \times 37$

$f(x_0) = 52 - 4.12 = 47.88$

$f(x_0) = 48$

Hence 48 students obtained less than 45 marks

Ques 2 find the value of $\sin 52°$ from the given table

θ	45°	50°	65°	60°
$\sin\theta$	0.7071	0.7660	0.8192	0.8660

Ans 0.988[illegible]32